1. (a) Express $\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x$ as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x = \ln k$$

	$\underset{x=2.1}{\overset{\alpha x\to 0}{\longrightarrow}} X$	
where k is a constant to be found.	X-2.1	(2)
		(2)

2.	In this question you must show all stages of your working.
	Solutions relying on calculator technology are not acceptable.

Show that

$$\int_1^{e^2} x^3 \ln x \, \mathrm{d}x = a \mathrm{e}^8 + b$$

where a and b are rational constants to be found.	(5)

The table below shows corresponding values of x and y for $y = log_3 2x$

The values of y are given to 2 decimal places as appropriate.

х	3	4.5	6	7.5	9
у	1.63	2	2.26	2.46	2.63

(a) Using the trapezium rule with all the values of y in the table, find an estimate for

$$\int_{3}^{9} \log_3 2x \, \mathrm{d}x$$

(3)

Using your answer to part (a) and making your method clear, estimate

(b) (i)
$$\int_{3}^{9} \log_{3} (2x)^{10} dx$$
(ii)
$$\int_{3}^{9} \log_{3} 18x dx$$

(ii)
$$\int_{3}^{9} \log_3 18x \, \mathrm{d}x$$

(3)

4.

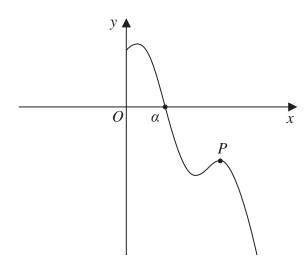


Figure 2

Figure 2 shows a sketch of part of the curve with equation y = f(x) where

$$f(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$$

and *x* is measured in radians.

The point *P*, shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P, giving your answer to 3 significant figures.

(4)

The curve crosses the x-axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, f(4) = 4.274 and f(5) = -1.212

(b) explain why α must lie in the interval [4, 5]

(1)

(c) Taking $x_0 = 5$ as a first approximation to α , apply the Newton-Raphson method once to f(x) to obtain a second approximation to α .

Show your method and give your answer to 3 significant figures.

(2)

5.	$f(x) = \frac{3kx - 18}{(x+4)(x-2)}$	where k is a positive constant
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- (a) Express f(x) in partial fractions in terms of k. (3)
- (b) Hence find the exact value of k for which

$$\int_{-3}^1 \mathbf{f}(x) \, \mathrm{d}x = 21$$

(4)

6.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(3+x)^{-2}$$

writing each term in simplest form.

(4)

(b) Using the answer to part (a) and using algebraic integration, estimate the value of

$$\int_{0.2}^{0.4} \frac{6x}{(3+x)^2} \, \mathrm{d}x$$

giving your answer to 4 significant figures.

(4)

(c) Find, using algebraic integration, the exact value of

$$\int_{0.2}^{0.4} \frac{6x}{(3+x)^2} \, \mathrm{d}x$$

giving your answer in the form $a \ln b + c$, where a, b and c are constants to be found.

(5)
